MATH 1450 EXAM 2

NAME_____ **GRADE** _____-OUT OF **15 PTS**

Answer each of following questions correctly for a full credit.

1. (**4.5pts**) **Related Rates:** Answer <u>each</u> question $(a-c)$ separately (up to three decimal places.)

   (a) A hot air balloon rising vertically is tracked by an observer located 4km from the lift-off point. At a certain moment, the angle between the observer's line of sight and the horizontal is $\frac{\pi}{5}$ and it is changing at a rate of 0.4 rad/hr. How fast is the balloon rising at this moment?

   (b) Water pours into a conical tank of height 10 m and radius 4 m at a rate of 6 m^3/min. (i) At what rate is the water level rising when the level is 5 m high? (ii) As time passes, what happens to the rate at which the water level rises? (Recall: $V = \frac{1}{3}\pi r^2 h$)

(c) A particle is moving along the curve $y = \sqrt{x}$. As the particle passes through the point $(4, 2)$, its x–coordinate is increasing at a rate of 3 cm/sec. How fast is the distance from the particle to the origin changing at this instant? (up to *three decimals*).

2. (**4.5pts**) **Approximation and Newton's method**:

(a) Compute the linearization of $f(x) = e^{x-1}$ at $c = 1$.

(b) Calculate (up to *five decimals*) the first three approximations x_1, x_2, x_3 to a root of $g(x) = x^2 - 5$ using the initial guess $x_0 = 2$.

(c) Find the linear approximation to $f(x) = \ln(x)$ near 1.

3. (**6pts**) **First and Second Derivatives Tests**

(a) Find the open intervals where $g(x) = x + 2sin(x)$ is monotonic, i.e., increasing or decreasing, with $0 < x < 2\pi$. (**No** decimal)

(b) Find all critical and the extreme values of the function $f(x) = 2x^3 - 9x^2 + 12x$ on the interval $[0, 3]$.

(c) Answer the following questions for the given function $f(x) = 3x - x^3$.

i- Find the critical *points*.

ii- Find where (*in an interval form*) the function is increasing / decreasing.

iii- Find the relative maxima and minima, if any.

iv- Find where (*the value(s) of* x) the second derivative is zero or undefined.

v- Find where (*in an interval form*) the function is concave up / down.

vi- Find the inflection *point(s)*, if any.